Sea ice is easy:

d
-- ***Ice mass = Freezing - Melting + Import - Export***
dt

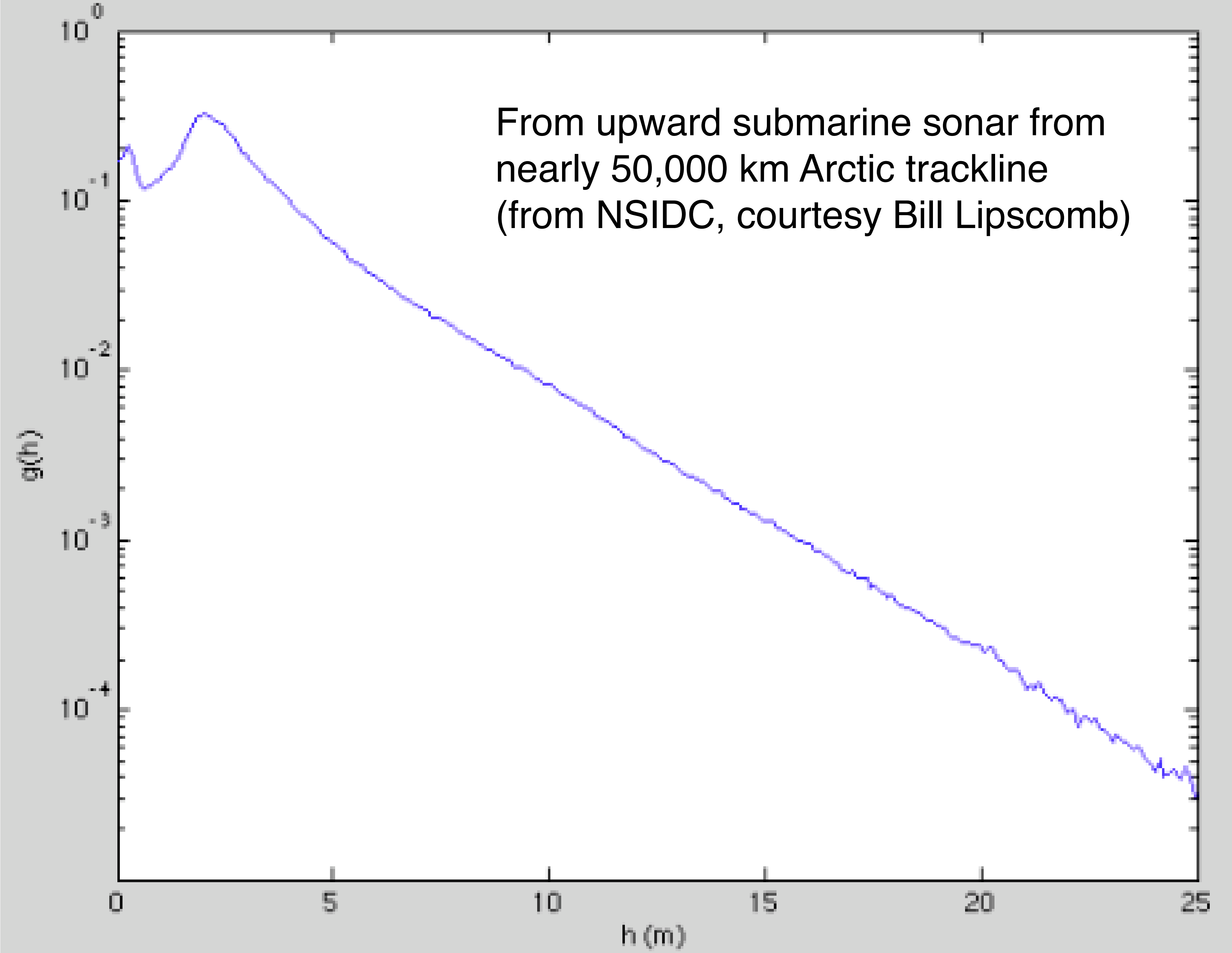
The only hard parts are

Freezing, ***Melting,*** ***Import,*** *and* ***Export***

Import & Export are complicated because you need *ice motion*.

Freezing & Melting are complicated because it's not enough to know only average thickenss.

-- You need ***probability distribution of thickness***.


From upward submarine sonar from
nearly 50,000 km Arctic trackline
(from NSIDC, courtesy Bill Lipscomb)
g(h)
h (m)
10^0
10^-1
10^-2
10^-3
10^-4
0
5
10
15
20
25


Santa
work
shop

A (very!) little math:

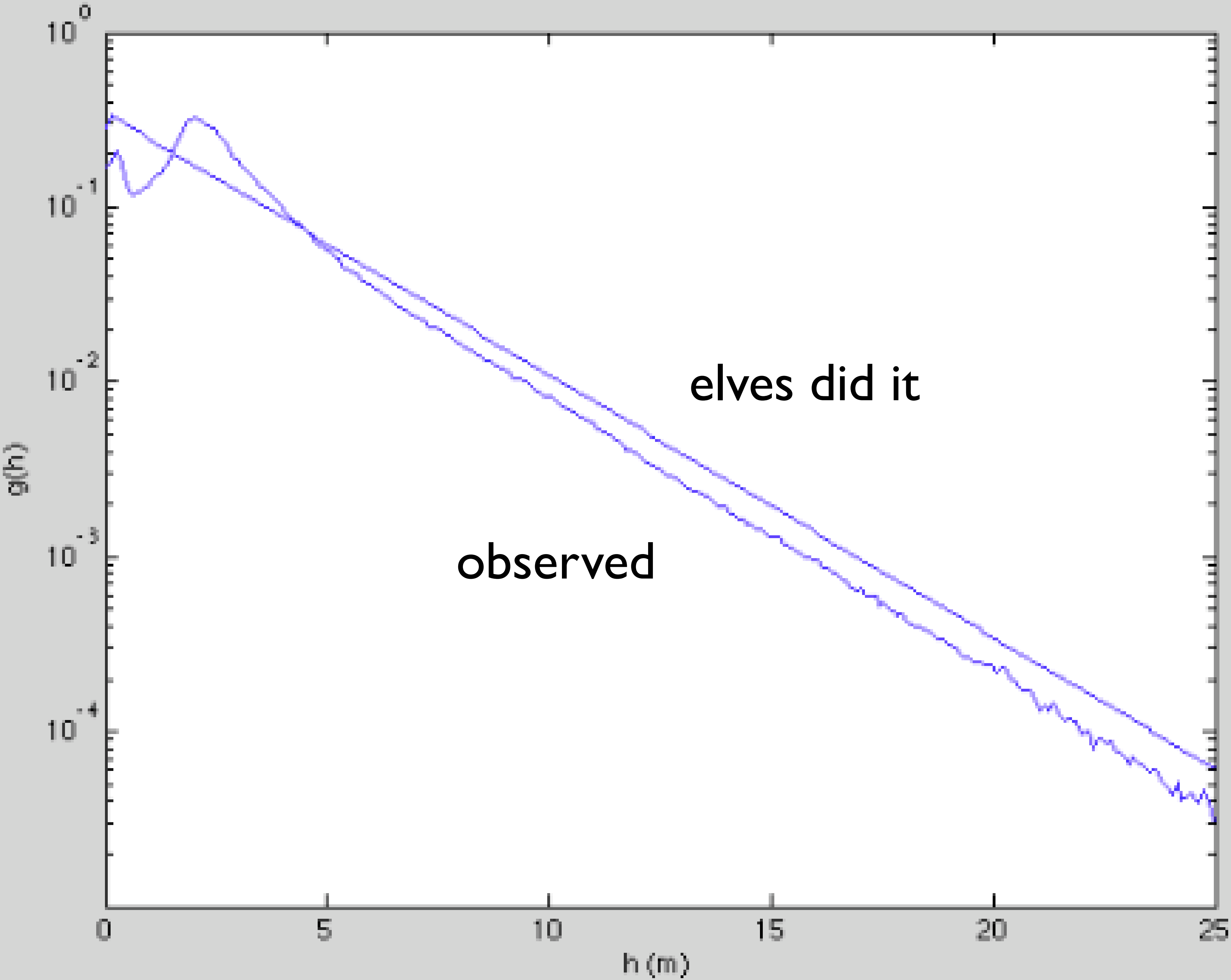
Maximise $S = -\int_0^\infty \log(g) g\,dh$

subject to $\int_0^\infty g\,dh = 1$ and $\int_0^\infty gh\,dh = H$

and *presto!* $g(h) = \exp(-h/H) / H$

Thus, Santa's elves, underoccupied during off-season, randomly toss ice into the Arctic. Is that easy, or what?

Let's see!



elves did it
observed
g(h)
h (m)
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
5
10
15
20
25

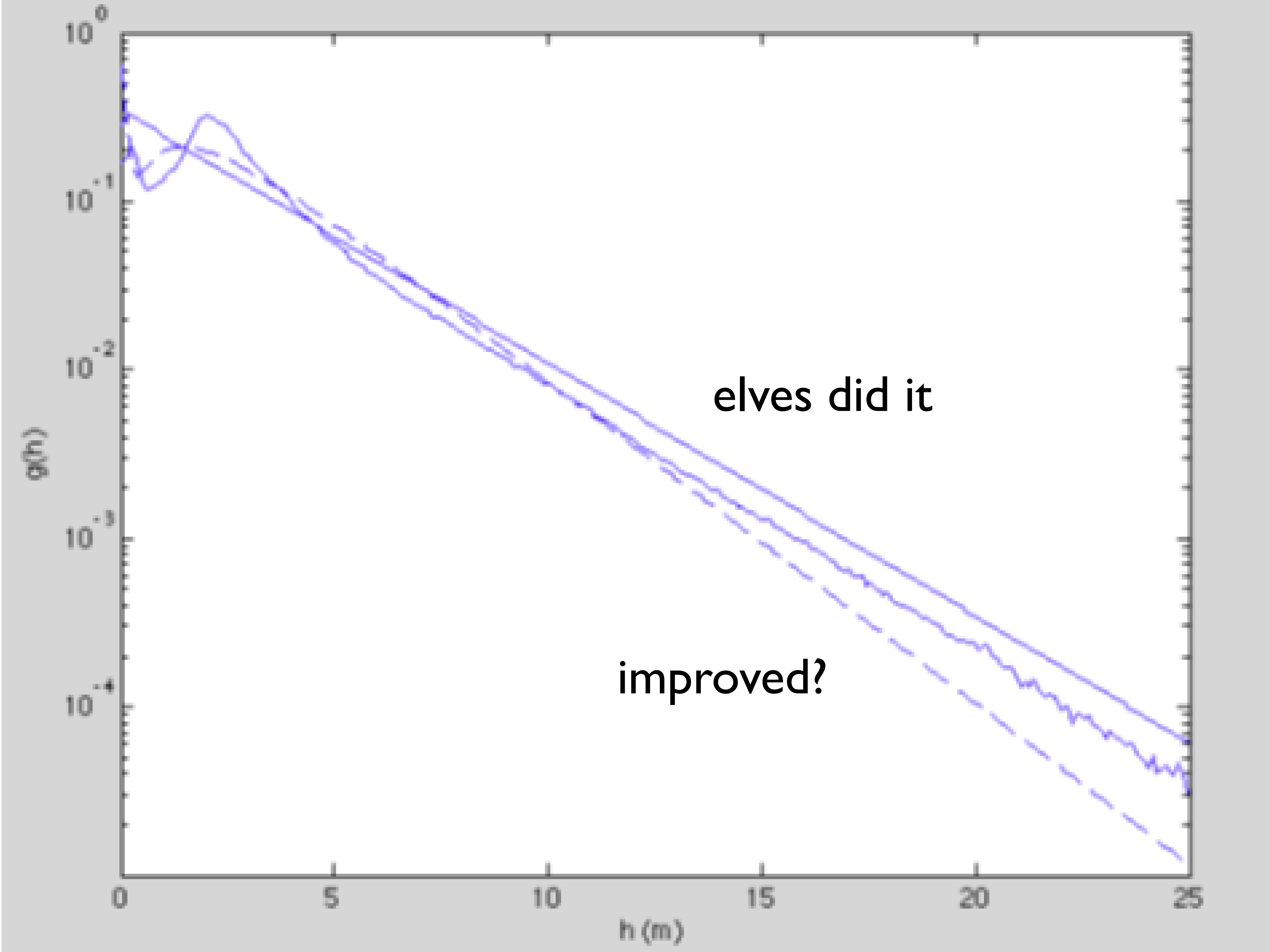
OK, not the greatest ever. But fiddle just a little.

Wind shifts, tides, inertial oscillations, ... open ice.

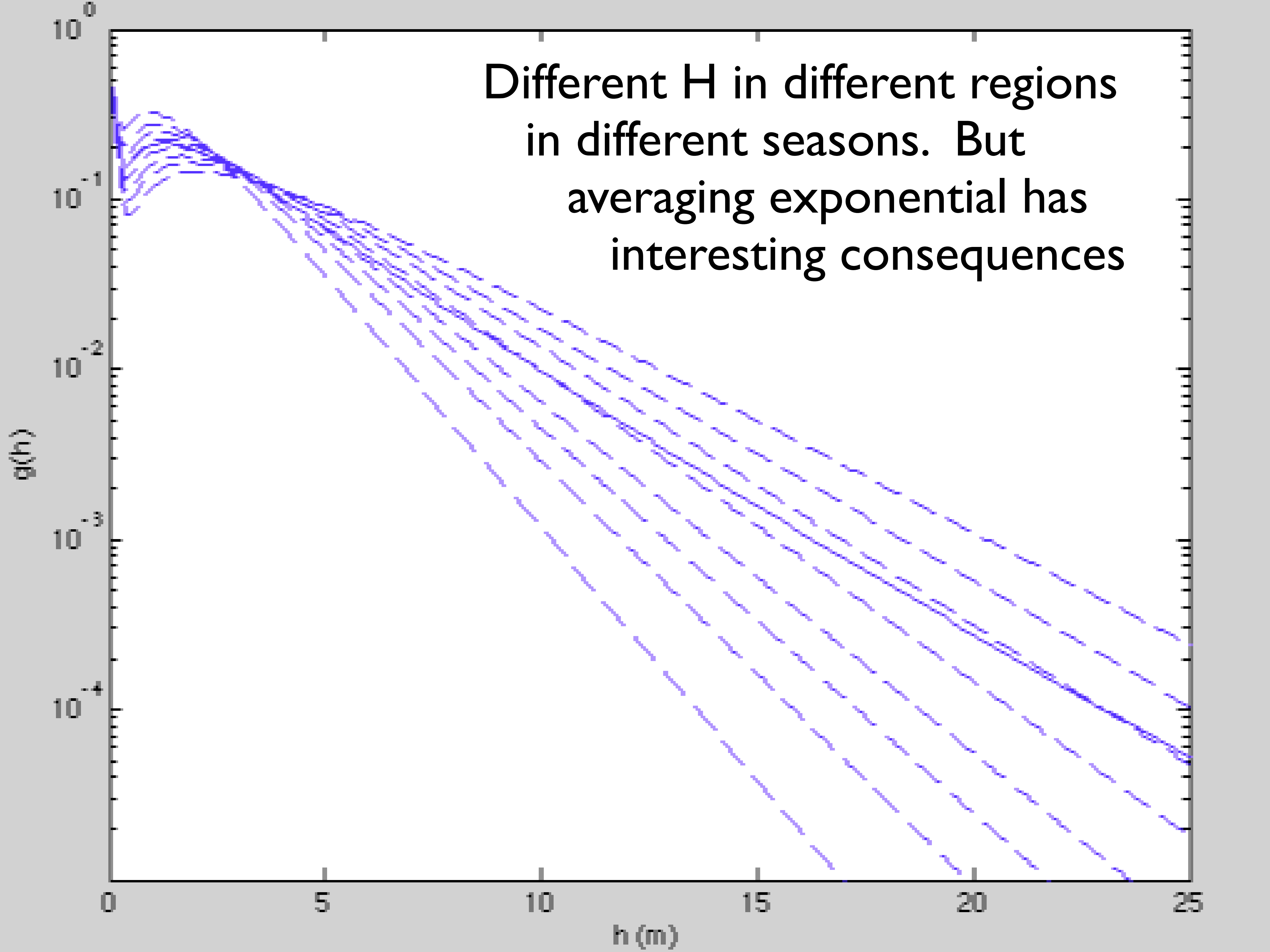
Recognize an "open water" (thin ice?) fraction 1-*A*. In leads during freezing, thin new ice rapidly forms but is easily smunched into thicker ice. Fix it?

$$D = \frac{A}{(2a-1)H}\left(e^{-\frac{h}{aH}} - e^{-\frac{h}{H-aH}}\right) + \frac{1-A}{bH}e^{-\frac{h}{bH}}$$

Improved? Let's see.



elves did it
improved?
g(h)
h (m)
0
5
10
15
20
25


Different H in different regions in different seasons. But averaging exponential has interesting consequences
g(h)
h (m)
10^0
10^-1
10^-2
10^-3
10^-4
0
5
10
15
20
25

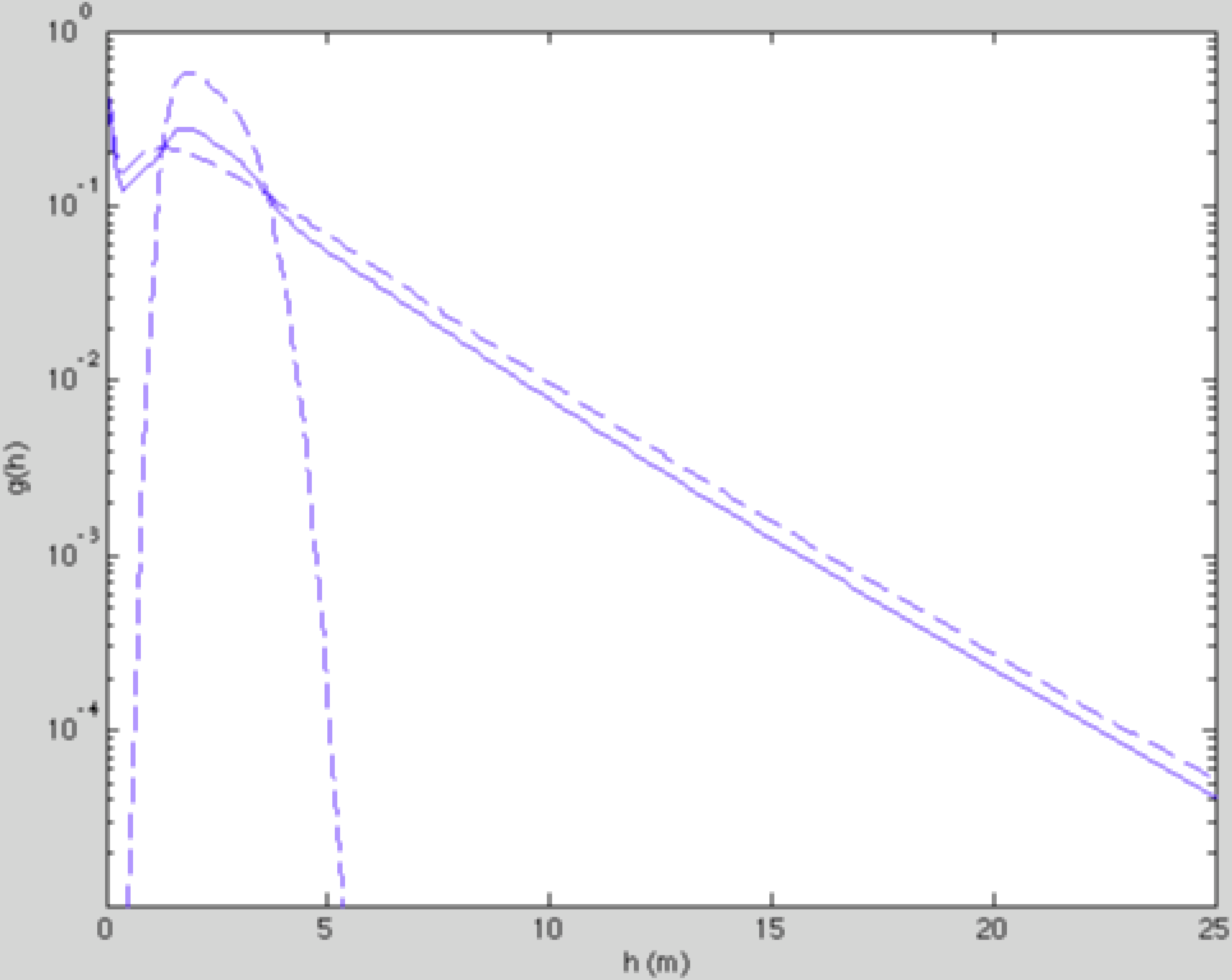
Dynamics -- alone -- spreads out *g(h)*, approaching *D.*

Thermodynamics -- alone -- would focus *g(h)*, e.g. as

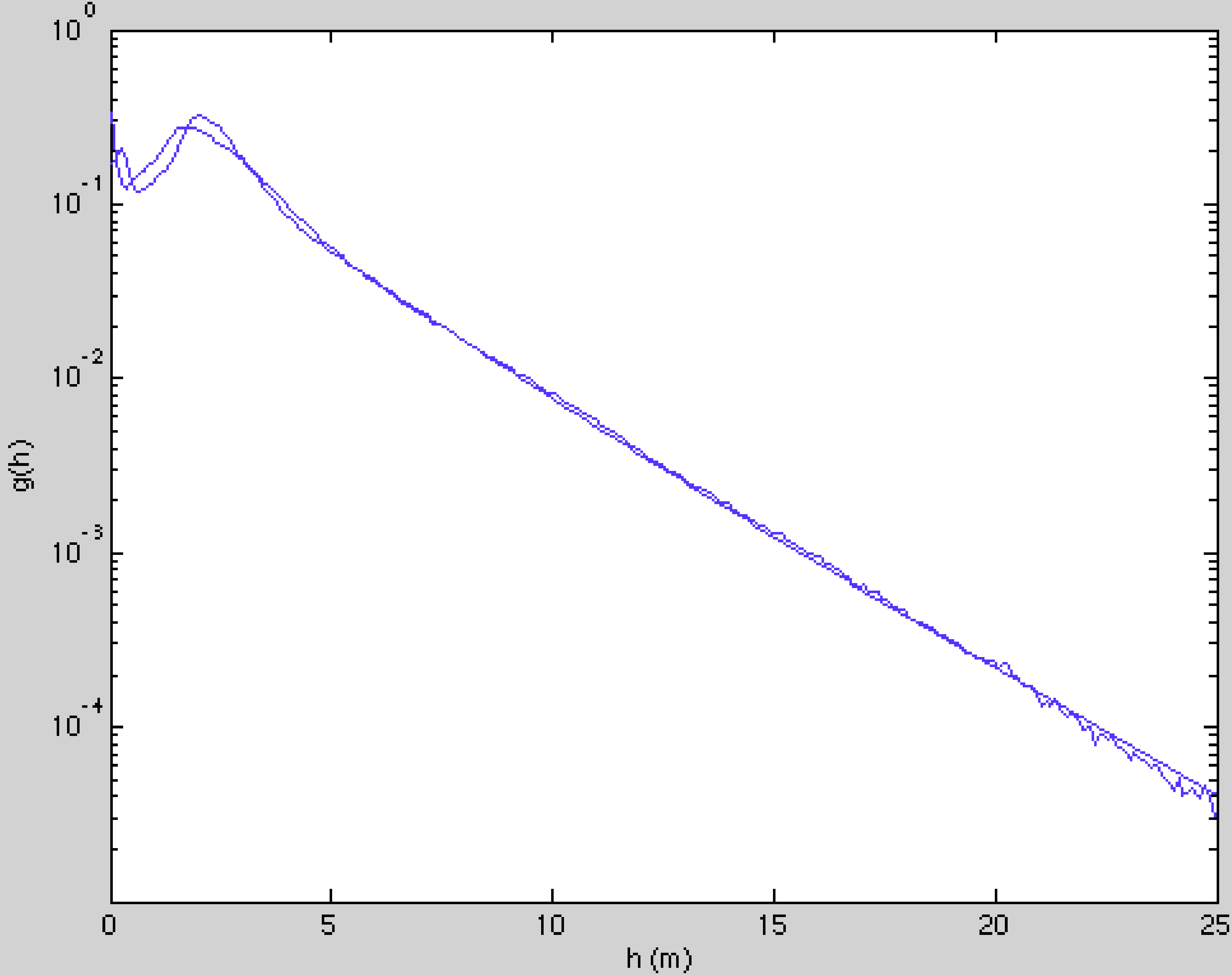
$$T = e^{-\frac{(h-aH)^2}{2b^2}} \Big/ \sqrt{2\pi b}$$

which also needs be averaged over various *H*. Combine to $g(h) = cD + (1-c)T$ for some *c.*

See a case *c* =0.8:



10^0
10^-1
10^-2
10^-3
10^-4
g(h)
0
5
10
15
20
25
h (m)



10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
g(h)
0
5
10
15
20
25
h (m)

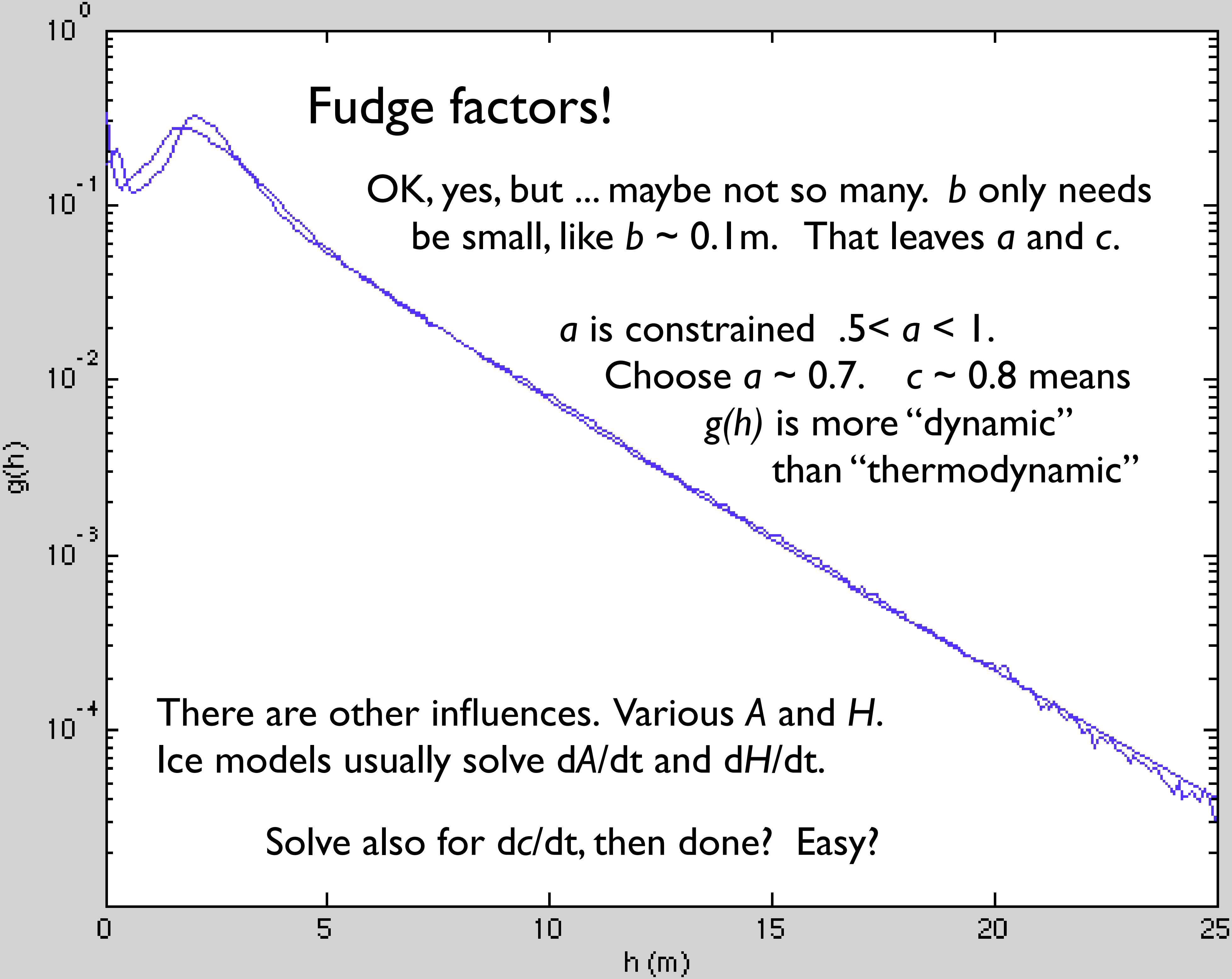
Fudge factors!

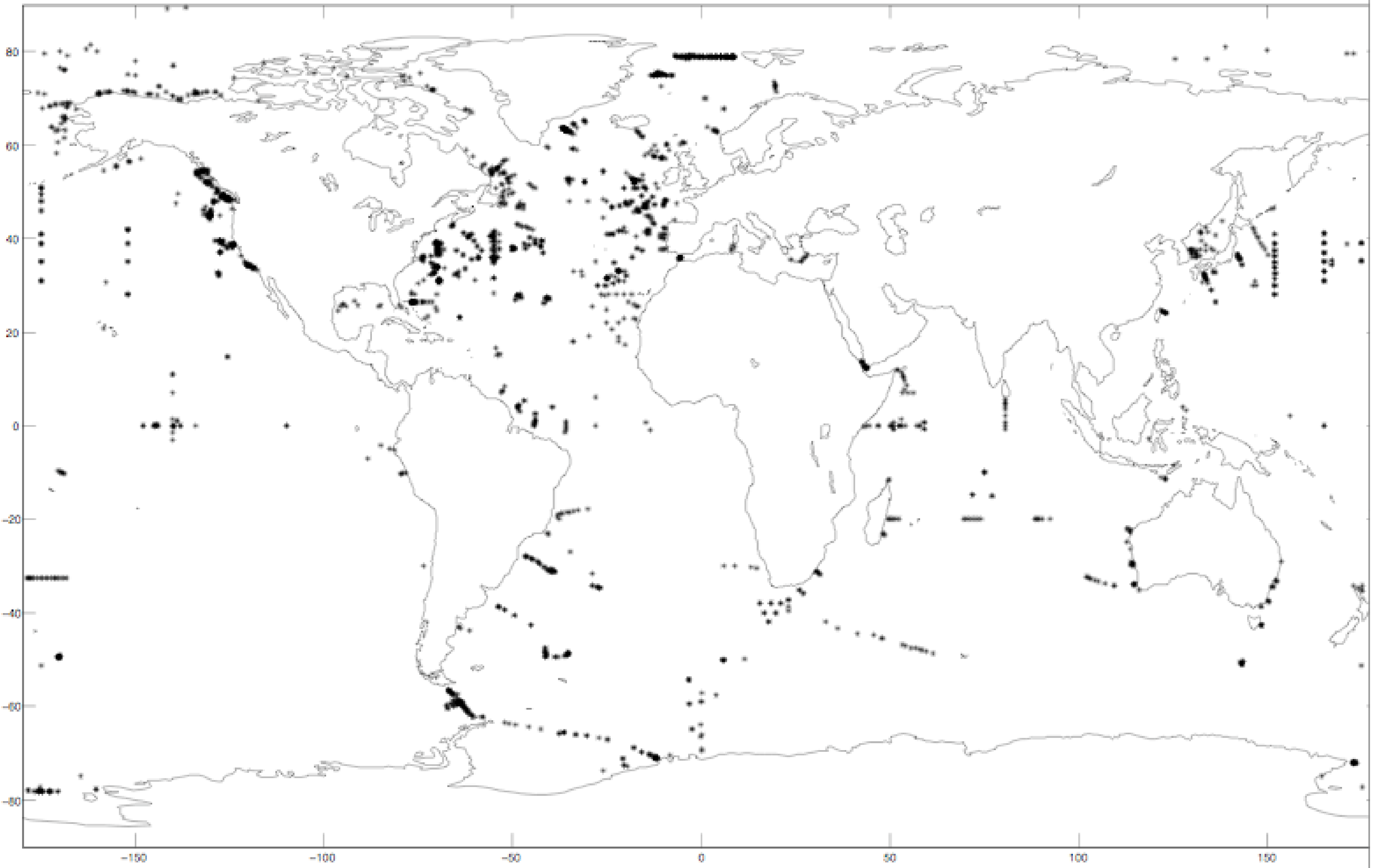
OK, yes, but ... maybe not so many. *b* only needs be small, like *b* ~ 0.1m. That leaves *a* and *c*.

a is constrained .5< *a* < 1. Choose *a* ~ 0.7. *c* ~ 0.8 means *g(h)* is more "dynamic" than "thermodynamic"

There are other influences. Various *A* and *H*. Ice models usually solve d*A*/dt and d*H*/dt.

Solve also for d*c*/dt, then done? Easy?





12856 CM records, 81669 months later: “topostrophy” **fxV·S**=+.24

Why “+”? Why “.24”?